Class –T.Y.B.Sc

Semester-VI

Paper-III – Topology of Metric spaces and Real Analysis

Sample Questions

		Choose correct alternative in each of the following		
1		Let $A = \{x \in \mathbb{R} / \cos x \neq 1\}$, the distance in $\mathbb{R}$ being usual. Then,		
	(a)	A is an infinite open set	(b)	A is a finite closed set
	(c)	A is an open set	(d)	A is neither open nor closed
2		Let $f: \mathbb{R}^n \to \mathbb{R}$ be defined by $f(x) = \|x\|$ then where $\|x\|$ represents the Euclidean norm		
	(a)	f is continuous but not uniformly continuous on $\mathbb{R}^n$	(b)	f is not continuous on $\mathbb{R}^n$
	(c)	f is uniformly continuous on $\mathbb{R}^n$	(d)	Nothing can be said about f
3		$f: \mathbb{R} \to \mathbb{R}$ be continuous function with respect to usual metric, then $f^{-1}(0, \infty)$ is		
	(a)	Open set in $\mathbb{R}$	(b)	Closed set in $\mathbb{R}$
	(c)	Bounded set in $\mathbb{R}$	(d)	Compact set in $\mathbb{R}$
4		Which of the following real valued functions are uniformly continuous on the give sets. (i) $f(x) = \sqrt{x}$ on $[0,1]$ (ii) $f(x) = x^3$ on $\mathbb{R}$		
	(a)	only (i)	(b)	only (ii)
	(c)	both (i) and (ii)	(d)	Neither (i) nor (ii)
5		Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^2 + 3y^2$. Then		
	(a)	f is not continuous at $(x, 0)$ for any $x \in \mathbb{N}$.	(b)	f is not continuous at $(0, 0)$
	(c)	f is continuous at $\mathbb{R}^2$	(d)	f is not continuous at finitely many points in $\mathbb{R}^2$
6		If Let $f: (0,1) \to [0,1]$ is a bijective function, then		
	(a)	f is not continuous on $(0, 1)$	(b)	f is continuous but not uniformly continuous on$(0, 1)$
	(c)	f is uniformly continuous on$(0, 1)$	(d)	Nothing can be said about the continuity of f
7		Let (X, d) be a discrete metric space and (Y, d_0) be any metric space. If $f : X \to Y$, then f is		
	(a)	an uniformly continuous function on X	(b)	a bounded function on X.
	(c)	continuous but not uniformly continuous function on X	(d)	Discontinuous function on X
8		Which of the following statements is True in $\mathbb{R}^n$?		
	(a)	Continuous image of a closed set is closed.	(b)	Continuous image of a connected set need not be connected.
	(c)	Continuous image of a path connected set is path connected.	(d)	Continuous image of a open set is open.
9		$E = \{ (x, y) \in \mathbb{R}^2 \| y \neq x\}$ be subset of $\mathbb{R}^2$and d be the Euclidean metric, then		

	(a)	E is disconnected	(b)	E is connected
	(c)	E is path connected	(d)	E is connected but not path connected
10	If $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 81\}$ is a subset of $\mathbb{R}^2$ with Euclidean distance, then S is			
	(a)	compact and connected	(b)	compact but not connected
	(c)	connected but not compact	(d)	neither compact nor connected
11	In ($\mathbb{R}^2$,d) where d is Euclidean distance, the following set is not path connected.			
	(a)	$\mathbb{R}^2 \setminus \{(0,0)\}$	(b)	$\mathbb{R}^2 \setminus \{B((1,0), 1)\}$
	(c)	$\mathbb{R}^2 \setminus \{(x, y) : y = 0\}$	(d)	$\{(x, y) : x^2 + y^2 = 2\}$
12	Let $A \subseteq \mathbb{Q}$. If A is a connected subset of $(\mathbb{R}, d)$ where d is usual distance then			
	(a)	$A = \mathbb{Q}$	(b)	A is an infinite bounded set
	(c)	A is a singleton set	(d)	A is finite bounded set containing more than one element
13	Consider the following subsets of $(\mathbb{R}^2, \mathrm{d})$ where d Euclidean. (i) Convex subset of $\mathbb{R}^2$ (ii) x axis (iii) $\{(\mathrm{x}, \mathrm{y}) \in \mathbb{R}^2 : \mathrm{xy} = 1\}$ Then,			
	(a)	(i), (ii), (iii) are all connected	(b)	(i), (ii) are connected
	(c)	Only (iii) is connected	(d)	only (i) is connected
14	Let (X, d) be a connected metric space and $f : X \to \mathbb{N}$ be a continuous map. Then			
	(a)	f is onto	(b)	f is one-one
	(c)	f is bijective	(d)	f is constant
15	Let $f_n : [0, 1] \to [0, 1]$ be defined by $f_n(x) = x * X_n(x)$ where $$X_n(x) = \begin{cases} 0 & if\ x \notin \left[0, \frac{1}{n}\right] \\ 1 & if\ x \in \left[0, \frac{1}{n}\right] \end{cases}$$			
	(a)	$\{f_n\}$ converges uniformly to 0 on $[0, 1]$.	(b)	$\{f_n\}$ converges uniformly to 1 on $[0, 1]$
	(c)	$\{f_n\}$ converges pointwise on $[0, 1]$ but does not converge uniformly.	(d)	$\{f_n\}$ does not converge pointwise on $[0, 1]$.
16	If $\{f_n\}$ and $\{g_n\}$are sequences of functions on $S, S \subseteq \mathbb{R}$ converging uniformly to f and g respectively on S then the following sequence of function may not converge uniformly of S to the given function			
	(a)	$\{f_n + g_n\}$to $f + g$	(b)	$\{f_n - g_n\}$ to $f - g$
	(c)	$\{\lambda\, f_n\}$ to λf	(d)	$\{f_n * g_n\}$ to $f * g$
17	$f_n(x) = x^n$ for $x \in [0, 1]$			
	(a)	The pointwise limit of $\{f_n\}$ is not continuous on $[0, 1]$	(b)	$\{f_n\}$ converges uniformly on [0,1] to a continuous function.
	(c)	$\{f_n\}$ converges pointwise on $[0, 1]$ to a continuous function	(d)	The pointwise limit of $\{f_n\}$ does not exist
18	The series$\sum_{n=0}^{\infty} \frac{x^n}{n+1}$ is			
	(a)	uniformly convergent on $\mathbb{R}$	(b)	not uniformly convergent on $[-a, a]\ where\ 0 < a < 1$
	(c)	uniformly convergent on $[-a, a]\ where\ 0 < a < 1$.	(d)	Not pointwise convergent on $[-a, a]\ where\ 0 < a < 1$
19	If R is the radius of convergence of power series $\sum_{n=0}^{\infty} c_n x^n$, then radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{5n}$is			
	(a)	R^5	(b)	R

	(c)	$R^{\frac{1}{5}}$	(d)	R^6
20	Let $a \in \mathbb{R}, a > 1$ The series $\sum_{n=0}^{\infty} f_n(x)$ where $f_n(x) = \frac{1}{x^n+1}$, $x \in \mathbb{R}$			
	(a)	uniformly convergent on $[a, \infty)$.	(b)	pointwise convergent on [0,1).
	(c)	uniformly convergent on $[0, 1)$.	(d)	Not uniformly convergent on $[a, \infty)$.